Extended analysis of the "penguin" part of $K \to \pi\pi$ amplitude

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Abstract

We make an attempt to clarify the role of the annihilation or "penguin" mode in the description of the $K \to \pi\pi$ decay within the Standard Model. The attention is concentrated on new operators in the effective $\Delta S = 1$ Hamiltonian and the violation of factorization for mesonic matrix elements of the local four-quark operators. We propose a regular method to evaluate the mesonic matrix elements of $K \to \pi\pi$ transitions based on studying three-point correlators via QCD sum rules using the chiral effective theory as an underlying low-energy model for strong interaction. Matrix elements of the QCD penguin operator are calculated within this approach. The total "penguin" contribution is found to be relatively large that improves the theoretical description of the $\Delta I = 1/2$ rule in non-leptonic kaon decays.

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1 Introduction.

At present the Standard Model (SM) [1] of electroweak and strong interactions seems to fit successfully all known data of particle phenomenology and provides a good prototype of future theory of unification. Even though the SM itself is a little unsatisfactory from the aesthetic point of view, it plays an essential role as a convenient tool of compact description of all available experimental facts. However along with many achievements of the SM there exist several subtle points still to be understood. These points are of particular interest because their resolution might lead to a discovery of new features absent within the SM itself and could serve as an indication on new physics beyond the SM. Before claiming the discovery of new physics and demanding going beyond the SM, however, the corresponding phenomenon should be examined thoroughly to guarantee it cannot be explained at the level of the SM and the proper accuracy is achieved. Unfortunately this task cannot presently be easily accomplished.

One example of these subtle points within the SM is the origin for considerable enhancement of the $\Delta I = 1/2$ parts of the amplitudes in non-leptonic kaon decays. Though the $\Delta I = 1/2$ rule is explained qualitatively by the influence of strong interactions the detailed quantitative description is still lacking. The closely related problem of the "direct" CP violation in kaon decays also requires a more precise quantitative analysis.

Recently numerous attempts have been made to improve the theoretical description of non-leptonic kaon decays in SM with sufficient accuracy. The efforts are exerted both to extend the perturbative QCD analysis [2] beyond the leading order and to account more adequately for long-distance effects using some models of strong interactions at low energy such as, for example, chiral Lagrangians [3], $1/N_c$ expansion of many color QCD [4, 5] or lattice simulations [6]. However the present results still do not reproduce the experimental data and the existing accuracy is not satisfactory.

In the present paper we study some questions connected with the annihilation mode of the $K \to \pi\pi$ decay which were not considered in the previous analysis. This mode is important in explanation of the $\Delta I = 1/2$ rule because it is purely of the $\Delta I = 1/2$ sort

and its presence is the main distinction between $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes at least within the perturbation theory. It also generates the imaginary part of the effective Hamiltonian and, therefore, is responsible for the non-zero value of the ϵ' parameter describing the direct CP violation in kaon decays

At the fundamental level of the SM the strangeness changing transitions with $\Delta S = 1$ occur via the W-boson exchange between two weak charged currents. The short-distance analysis of the product of weak hadronic currents after removing the W-boson and the heavy quarks results in an effective $\Delta S = 1$ Hamiltonian of the following form [7, 8, 9]

$$H_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_{i=1}^{6} [z_i(\mu) + \tau y_i(\mu) Q_i] + h.c. \tag{1.1}$$

Here G_F is the Fermi constant, V stands for the quark flavor mixing matrix, $\tau = -V_{td}V_{ts}^*/V_{ud}V_{us}^*$, $z_i(\mu)$ and $y_i(\mu)$ are the coefficients of the Wilson expansion subtracted at the point μ and $\{Q_i | i = 1, ..., 6\}$ is the basis of the local operators containing light quark fields (u, d, s) only with dimension six in mass units (four-quark operators). In eq. (1) we have omitted the contributions of electroweak penguin operators [9, 10]. The current-current operators $\{Q_i | i = 1, 2\}$ do not form a close set under renormalization and additional so-called QCD penguin operators $\{Q_i | i = 3, ..., 6\}$ are produced by an annihilation diagram. Since the discovery of the annihilation mode [7] it was widely discussed in the literature. The present analysis however cannot be considered a complete one. Following points require further investigation:

1. The QCD perturbation theory analysis of the kaon non-leptonic decay can be improved by using more accurate effective Hamiltonian. In the standard approach only the leading terms in the inverse masses of heavy quarks are taken into consideration while a proper account of the non-leading corrections in the inverse mass of charmed quark generated by the penguin-type diagrams along with the usual expansion in the strong coupling constant α_s can be important. Analogous corrections to the $K^0 - \bar{K}^0$ mixing have been considered in ref. [11] and turn out not to be negligible though their actual magnitude can strongly depend on the procedure used for estimation of the corresponding matrix element. Further corrections to the effective Hamiltonian appear because the top quark is heavier than other

quarks and W-boson. This results in an incomplete GIM cancellation of the annihilation diagram including t-quark. Though this question was widely discussed [9] the appearance of a new operator in the effective Hamiltonian remained beyond consideration.

2. An achievement of a high precision in theoretical estimates for the kaon decays stumbles at a necessity to calculate mesonic matrix elements of local four-quark operators in the effective Hamiltonian (1.1). The only method of computation entirely based on first principles seems to be numerical simulations on the lattice though so far even this approach has not provided us with unambiguous estimates due to some subtleties connected with the description of fermions on the lattice. Meanwhile several semi-phenomenological techniques have been developed and applied for computation of those matrix elements, for example, the many color expansion of QCD [4, 5]. Their precision, however, still need to be essentially improved.

Recently a new regular method to evaluate the mesonic matrix elements has been proposed [12] where the effective Hamiltonian is represented in terms of the chiral effective theory variables and the parameters of the chiral representation are determined via QCD sum rules for an appropriate three-point Green's function. In Sect. 3 we briefly describe the method taking as an example the calculation of the QCD penguin operator matrix elements which has been a subject of a few controversies [4, 5, 7, 13, 14]. This technique will be applied in Sect. 4 to estimate the scalar gluonium contribution to the $K \to \pi\pi$ decay amplitude.

3. The renormalization group improved perturbation theory does not take into account the strong interaction of the soft light quarks and gluons with the virtual momentum smaller than the normalization point $\mu \sim 1\ GeV$. The information on this interaction is entirely contained in the mesonic matrix elements of the local four-quark operators. Well known factorization procedure for evaluation of these matrix elements [15] accounts only for the "factorizable" part of the interaction [16]. "Unfactorizable" contributions, for example, those corresponding to the annihilation of a quark pair from the four-quark operator into soft gluons are omitted when the factorization procedure is performed. The

calculation of these contributions and the generalization of the matrix elements beyond the factorization framework can be systematically done within the approach applied in Sect. 3 to the calculation of the penguin operator matrix element. It turns out to be possible to obtain an information on the important part of the factorization violating contribution to the $K \to \pi\pi$ decay amplitudes. This possibility is connected with the investigation of a new $K \to \pi\pi$ decay channel induced by annihilation of the quark pair from the four-quark operator into gluons with the subsequent formation of the pion pair by the soft gluon cloud, *i.e.* the decay channel with the gluons playing the role of the intermediate state [17]. Being unfactorizable this decay mode does not appear as a correction to some leading order contribution and can be studied by its own. This feature makes obtained results more accurate.

In the Sect. 4 we study a new $K \to \pi\pi$ decay channel with the simplest scalar colorless gluon configuration forming an intermediate state. We calculate both short-distance (perturbative) and long-distance (non-perturbative) part of the corresponding amplitude. The short-distance analysis is based on the results of the Sect. 2. To obtain the long-distance contribution the chiral Lagrangians are used as a low-energy model of strong interactions and corresponding chiral coupling is derived via QCD sum rules.

2 The local effective $\Delta S = 1$ Hamiltonian beyond the four-quark approximation.

We begin with the calculation of the non-leading charmed quark mass correction. Expressions for Wilson coefficients $z_i(\mu)$ and $y_i(\mu)$ in eq. (1.1) in the region of QCD asymptotic freedom are obtained by performing the operator product expansion (OPE) of two weak charged quark currents. At a typical energy scale of weak decays of light hadrons these Wilson coefficients have been with the renormalization group technique. After removing all heavy particles (W-boson, t-, b-, and c-quarks) from the light sector of the theory formula (1.1) corresponds to the leading order in inverse masses of these particles. The removal of the c-quark however is not very reliable and, in general, requires a special in-

vestigation it is not heavy enough in comparison with the characteristic mass scale in the sector of light u-, d-, and s-quarks, for example, with the ρ-meson mass. The non-leading terms in the $1/m_c$ expansion can, therefore, be important and require a quantitative consideration.

The effective low-energy tree level Hamiltonian for $\Delta S = 1$ transitions before decoupling the c-quark reads

$$H^{tr}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} (Q^u_2 - (1-\tau) Q^c_2) + h.c. \tag{2.1}$$

where $Q^q_2 = 4(\bar{s}_L \gamma_\mu q_L)(\bar{q}_L \gamma_\mu d_L)$, $q_{L(R)}$ stands for left-(right-) handed quark. Performing the OPE and restricting oneself to the first order terms in α_s and m_c^{-2} one can write the representation for the effective Hamiltonian in the form

$$H_{\Delta S=1} = H^{(6)} + H^{(8)}. \tag{2.2}$$

The first addendum on the right-hand side (*rhs*) of eq. (2.2), $H^{(6)}$, corresponds to the leading contributions in $1/m_c$ and coincides with the *rhs* of eq. (1.1). Second addendum on the *rhs* of eq. (2.2), $H^{(8)}$, is the leading order $1/m_c$ correction [18]

$$H^{(8)} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} (1-\tau) \frac{\alpha_s}{4\pi} \left(\sum_{i=1}^{7} C^{(8)}_i Q^{(8)}_i + \sum_{i=1}^{4} C^{(7)}_i m_s Q^{(7)}_i \right) + h.c. \tag{2.3}$$

where a basis $\{Q^{(8)}_i | i = 1, ..., 7\}$ ($\{Q^{(7)}_i | i = 1, ..., 4\}$) of the local operators with dimension eight (seven) in mass units is chosen in the form

$$Q^{(8)}_1 = \bar{s}_L (\hat{D} G_{\mu\alpha} G_{\nu\mu} \sigma_{\alpha\nu} + G_{\nu\mu} \sigma_{\alpha\nu} \hat{D} G_{\mu\alpha}) d_L,$$

$$Q^{(8)}_2 = i g_s \bar{s}_L (J_\mu \gamma_\alpha G_{\alpha\mu} - \gamma_\alpha G_{\alpha\mu} J_\mu) d_L,$$

$$Q^{(8)}_3 = \bar{s}_L (P_\alpha G_{\mu\alpha} \gamma_\nu G_{\nu\mu} + \gamma_\nu G_{\nu\mu} G_{\mu\alpha} P_\alpha) d_L,$$

$$Q^{(8)}_4 = g_s \bar{s}_L (G_{\mu\nu} \sigma_{\mu\nu} \hat{J} + \hat{J} G_{\mu\nu} \sigma_{\mu\nu}) d_L,$$

$$Q^{(8)}_5 = i \bar{s}_L (G_{\mu\nu} \sigma_{\mu\nu} \gamma_\alpha G_{\alpha\beta} P_\beta - P_\beta \gamma_\alpha G_{\alpha\beta} G_{\mu\nu} \sigma_{\mu\nu}) d_L,$$

$$Q^{(8)}_6 = \bar{s}_L (D^2 \hat{J}) d_L, \quad Q^{(8)}_7 = i \bar{s}_L (\hat{D} G_{\nu\mu} G_{\nu\mu} - G_{\nu\mu} \hat{D} G_{\nu\mu}) d_L,$$

$$Q^{(7)}_1 = \bar{s}_R (G_{\mu\nu} \sigma_{\mu\nu} G_{\alpha\beta} \sigma_{\alpha\beta}) d_L, \quad Q^{(7)}_2 = \bar{s}_R (G_{\mu\nu} G_{\nu\mu}) d_L,$$

$$Q_3^{(7)} = i\bar{s}_R(G_{\nu\alpha}G_{\alpha\mu}\sigma_{\nu\mu})d_L, \quad Q_4^{(7)} = \bar{s}_R(J_\mu P_\mu + P_\mu J_\mu)d_L. \tag{2.4}$$

Here $P_\mu = i\partial_\mu + g_s A_\mu$ is the momentum operator in the presence of the external gluon field $A_\mu \equiv A_\mu^a t^a$, t^a are the standard generators of the color group $SU(3)$, $G_{\mu\nu} \equiv G_{\mu\nu}^a t^a$ is the gluon field strength tensor, $J_\mu \equiv \sum_{q=u,d,s}(\bar{q}\gamma_\mu t^a q)t^a$ and $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. When eq. (2.3) is derived we consider u- and d-quark to be massless, keep the first order quantities in strange quark mass and use the equations of motion

$$\bar{s}\hat{P} = m_s\bar{s}, \quad \hat{P}d = 0,$$

$$[P_\mu, G_{\mu\nu}] = iD_\mu G_{\mu\nu} = -ig_s J_\nu. \tag{2.5}$$

Straightforward calculations give the following numerical values for the coefficients $C_i^{(j)}$ to the leading order in the strong coupling constant α_s

$$C_1^{(8)} = \frac{8}{15}, \quad C_2^{(8)} = -\frac{16}{15}, \quad C_3^{(8)} = -\frac{4}{5}, \quad C_4^{(8)} = \frac{2}{15},$$
$$C_5^{(8)} = 0, \quad C_6^{(8)} = -\frac{8}{15}, \quad C_7^{(8)} = -\frac{2}{15},$$
$$C_1^{(7)} = -\frac{2}{5}, \quad C_2^{(7)} = -\frac{2}{5}, \quad C_3^{(7)} = \frac{6}{5}, \quad C_4^{(7)} = 0. \tag{2.6}$$

Thus we have generalized the effective Hamiltonian for $\Delta S = 1$ decays beyond the leading order in $1/m_c$.

To complete our treatment of the local effective Hamiltonian the annihilation of the top quark should be also considered. Because the top quark is heavy enough with respect to the W-boson [19] these two particles should be integrated out simultaneously to obtain the local effective Hamiltonian. The procedure of decoupling the heavy top quark is described in details in ref. [9] but one point has remained beyond the analysis. It is connected with other penguin-type operator $m_s Q^{(5)} = m_s\bar{s}_R g_s G_{\mu\nu}\sigma^{\mu\nu}d_L$. This quark-gluon operator was omitted in the early papers [7, 8] because the corresponding coefficient function vanishes in the first order in α_s. The next-to-leading calculations [20] has verified the smallness of this Wilson coefficient. However the above statement holds only under the assumption $m_t \ll M_W$, *i.e.* in the leading order in m_q^2/M_W^2 when the complete GIM cancellation takes place. In the case of a heavy top quark it becomes invalid and the quark-gluon

operator $m_sQ^{(5)}$ appears in the effective Hamiltonian already in the first order in α_s. This additional contribution has been obtained in ref. [21] and reads

$$\Delta H^{(6)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \tau C^{(5)}(\mu) m_s Q^{(5)}(\mu),$$

$$C^{(5)}(\mu) = \frac{1}{16\pi^2} \left(F(x_c) - F(x_t)\right) \eta(\mu), \quad x_q = \frac{m_q^2}{M_W^2},$$

$$F(x_q) = \frac{1}{3} \frac{1}{(x_q - 1)^4} \left(\frac{5}{2} {x_q}^4 - 7{x_q}^3 + \frac{39}{2} {x_q}^2 - 19x_q + 4 - 9{x_q}^2 \ln x_q \right),$$

$$F(x_c) \sim F(0) = \frac{4}{3}. \tag{2.7}$$

The renormalization group factor

$$\eta(\mu) = \left(\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_W)} \right)^{\gamma^{(5)}/2\beta_5} \left(\frac{\bar{\alpha}_s(m_c)}{\bar{\alpha}_s(m_b)} \right)^{\gamma^{(5)}/2\beta_4} \left(\frac{\bar{\alpha}_s(\mu)}{\bar{\alpha}_s(m_c)} \right)^{\gamma^{(5)}/2\beta_3}$$

where $\gamma^{(5)} = -28/3$ is the anomalous dimension of the operator $m_sQ^{(5)}$ [22], $\beta_{n_f} = 11 - \frac{2}{3}n_f$, n_f is the number of active quarks flavors, can be easily obtained because the operator $m_sQ^{(5)}$ does not mix with the four-quark operators in the leading order.

The contributions of the u- and c-quarks to the real part of the effective Hamiltonian cancel one another via GIM mechanism and the operator $m_sQ^{(5)}$ mainly contributes to the imaginary part of the effective Hamiltonian and, therefore, can be important in the analysis of direct CP violation. However, its contribution is suppressed numerically because $\eta(\mu) < 1$ and the function $F(x)$ changes slowly. Indeed, at the point $\Lambda_{QCD} = 0.3\ GeV$, $\mu = 1\ GeV$, $m_t = 130\ GeV$, we have $C^{(5)} = 0.0009$, while the numerical value of the same Wilson coefficient of the dominant penguin operator $Q_6 = -8\sum_{q=u,d,s}(\bar{s}_L q_R)(\bar{q}_R d_L)$ is $y_6 = 0.102$ [9].

Thus, the complete form of the effective Hamiltonian up to the first order in m_s, $1/m_c$ and α_s with $m_t \sim M_W$ becomes available now (eqs. (2.2, 2.3, 2.7)). The electroweak penguin operators have to be also included into the complete expression.

As an example of using the above Hamiltonian we consider the $K \to \pi\pi$ decays. For this end we have to extract an information about the matrix elements of the local operators $Q^{(j)}$ between the mesonic states. We start with the operators $m_sQ_i^{(7)}$ and $m_sQ^{(5)}$. These

operators contain explicitly the strange quark mass and, therefore, in the leading order of the chiral expansion they correspond to the tadpole term in the chiral weak Lagrangian. In detail the tadpoles will be considered in Sect. 3. Now we only note that tadpoles do not generate any observable effect and can be neglected in the leading order of the chiral symmetry breaking.

Here a remark about the role of the operator $m_s Q^{(5)}$ in determination of the parameter ϵ' is necessary. Recently an enhancement of the corresponding Wilson coefficient in the next-to-leading order in α_s has been discovered and a significance of this operator in the analysis of the direct CP violation was announced [23]. However in ref. [23] as in an earlier paper [21] the tadpole character of the operator $Q^{(5)}$ has not been recognized and its contribution to the parameter ϵ' was strongly overestimated. The correct treatment of this problem has appeared in the most recent paper [24] which is in agreement with our analysis.

Thus the problem is reduced to the estimation of the matrix elements of the operators $Q_i^{(8)}$. At present there is no regular method to calculate that kind of object within QCD excepting the numerical simulations on the lattice. To estimate at least the scale of the non-leading $1/m_c$ corrections we will work with the simplified model. As a first approximation we take the operators that survive after the factorization procedure. Then one selects the operators containing scalar quark currents which can be written as

$$(\bar{s}_L G_{\mu\nu}\sigma_{\mu\nu} q_R)(\bar{q}_R d_L) \quad \text{and} \quad (\bar{s}_L q_R)(\bar{q}_R G_{\mu\nu}\sigma_{\mu\nu} d_L).$$

This step seems to be justified because in the case of dimension six operators the similar "penguin-like" structures are strongly enhanced and dominate the others (see Sect. 3). The last simplification consists in the substitution

$$\bar{q} g_s G_{\mu\nu}\sigma_{\mu\nu} q \to m_0^2 \bar{q} q \tag{2.8}$$

where m_0 determines the scale of non-locality of the quark condensate and is defined by the equation $\langle \bar{q} g_s G_{\mu\nu}\sigma_{\mu\nu} q\rangle = m_0^2 \langle \bar{q} q\rangle, \quad m_0^2(1\ GeV) = 0.8 \pm 0.2\ GeV^2$ [25].

This substitution is valid in the chiral limit for the operator $\bar{q} g_s G_{\mu\nu}\sigma_{\mu\nu} q$ with dimension five in mass units. We suppose that it is justified also in our case at least for estimates up

to the order of magnitude. Actually all above assumptions as the factorization procedure in the case of dimension six operators become exact within the many color limit of QCD, $N_c \to \infty$, to the leading order in N_c.

Having adopted the assumptions described above the only operator which has the non-zero matrix element for the considered process is the operator $Q_4^{(8)}$ and the corresponding quantity reads

$$\langle \pi\pi | Q_4^{(8)} | K \rangle = \frac{m_0^2}{4} \langle \pi\pi | Q_6 | K \rangle. \tag{2.9}$$

We should note that the coefficients $C_i^{(8)}$ are finite to the leading order of the α_s and independent of renormalization scheme. We can therefore use the leading order values of the mesonic matrix elements that is consistent up to the used level of accuracy. In the contrast, the next-to-leading α_s corrections to Wilson coefficients depends on the renormalization scheme and to make the physical amplitudes scheme independent, matching between Wilson coefficients and mesonic matrix elements in the same renormalization scheme has to be made [2].

Thus, taking into account the first order $1/m_c$ corrections is reduced to the effective shift of the coefficients in front of the penguin operator Q_6 in the effective Hamiltonian (1.1)

$$z_6 \to \left(z_6 + \frac{\alpha_s}{4\pi} \frac{m_0^2}{4m_c^2} C_4^{(8)} \right), \qquad y_6 \to \left(y_6 - \frac{\alpha_s}{4\pi} \frac{m_0^2}{4m_c^2} C_4^{(8)} \right). \tag{2.10}$$

Using the numerical values $z_6 = -0.015$, $y_6 = -0.102$ at the point $\Lambda_{QCD} = 0.3\ GeV$, $\mu = 1\ GeV$, $m_t = 130\ GeV$ [9] one finds numerically the relative corrections to the Wilson coefficients in the form

$$z_6 \to z_6(1 - 0.1), \qquad y_6 \to y_6(1 + 0.01).$$

The main correction appears in the real part of the Wilson coefficient of the penguin operator Q_6. Parametrically, the contribution of the dimension eight operators can be as large as one half ($m_0^2/m_c^2 \sim 0.5$) of the one of dimension six operators, and not too small. In fact, we have found that when one estimates the mesonic matrix element of the local operator with dimension eight within the simplest factorization framework

the non-leading $1/m_c$ contributions to the kaon decay amplitudes are about 10% of the leading ones. However a large violation of the factorization for the matrix elements of the dimension eight operators does not seem to be impossible and the real value of the non-leading $1/m_c$ correction can be estimated only when somewhat self-consistent method to calculate these matrix elements within QCD will be available.

3 The QCD penguin operator contribution to the $K \to \pi\pi$ decay amplitude.

In this section we demonstrate a functioning of the method developed earlier in ref. [12] for calculating a contribution of the penguin operator Q_6 to the $K \to \pi\pi$ amplitude. This operator draws the special interest because its mesonic matrix element prevails over the current-current and other QCD penguin operators and gives a dominant contribution to the parameter ϵ' (we do not consider the electroweak penguins). Moreover, the estimates of matrix elements of the current-current operators are much less controversial and are quite reliably given by the simple factorization procedure. We leave aside technical details of the calculations that can be found in ref. [12] and focus on the main features of the approach.

The matrix element of the $K \to \pi\pi$ decay amplitude due to the operator Q_6 only reads

$$\langle \pi^+\pi^- | Q_6(\mu) | K^0 \rangle = \langle \pi^0\pi^0 | Q_6(\mu) | K^0 \rangle \equiv \langle \pi\pi | Q_6(\mu) | K^0 \rangle. \tag{3.1}$$

For comparison with other approaches we use the following parametrization of this matrix element

$$\frac{1}{i} \langle \pi\pi | Q_6(\mu) | K^0 \rangle = -B_p f_K m_K^2, \qquad f_K = 1.23 f_\pi, \; f_\pi = 132 \; MeV \; [19] \tag{3.2}$$

where the $f_K m_K^2$ factor fixes a natural mass scale while B_p is a dimensionless parameter to be computed.

We start with constructing the chiral effective theory describing weak and strong interactions of pseudoscalar mesons at low energy [3, 26]. The operator Q_6 is a composite quark operator which can be define accurately in the asymptotic freedom regime within

perturbation theory using the proper prescription of renormalization. Its matrix element can be hardly found in the practically interesting case of low energies where confinement takes place. However one can try to define the operator Q_6 at low energies within chiral perturbation theory, *i.e.* to find an effective realization of the operator Q_6 in terms of mesonic variables. The operator Q_6 belongs to the $8_L \times 1_R$ irreducible representation of chiral $SU(3)_L \times SU(3)_R$ group and has the following $SU(3)_V$ flavor quantum numbers: $S = 1,\ I = 1/2,\ I_3 = -1/2$. To the lowest order of a chiral perturbation theory expansion there are two Lorentz invariant bosonic operators with relevant properties $(\partial_\mu U^\dagger \partial^\mu U)_{23}$ and $(\chi^\dagger U + U^\dagger \chi)_{23}$ where χ stands for the pseudoscalar meson mass matrix and $U = e^{-i\sqrt{2}\phi/f_\pi}$ is the unitary matrix describing the octet of pseudoscalar mesons. Hence the operator Q_6 can be represented as

$$Q_6 = -f_\pi^4[g(\partial_\mu U^\dagger \partial^\mu U)_{23} + g'(\chi^\dagger U + U^\dagger \chi)_{23}] \tag{3.3}$$

where g and g' are the dimensionless parameters which are not fixed by symmetry requirements. After such an *ansatz* the mesonic matrix elements of the operator Q_6 become exactly computable within the effective theory, in other words, we have found a suitable kinematical framework for our dynamical problem which consists in determination of g and g'. For our aim we need the matrix elements

$$\langle \pi\pi | Q_6 | K^0 \rangle = 4igf_\pi m_K^2, \tag{3.4a}$$

$$\langle \pi^+(p_1) | Q_6 | K^+(p_2) \rangle = f_\pi^2(4g(p_1 p_2) + 4g' m_K^2), \tag{3.4b}$$

$$\langle 0 | Q_6 | K^0 \rangle = -4ig' f_\pi^3 m_K^2. \tag{3.4c}$$

We put the matrix element in eq. (3.4a) on the meson mass shell because this corresponds to the physical amplitude of $K \to \pi\pi$ transition. We also take into account that the term proportional to χ is equal to a full derivative due to equations of motion and does not contribute to the amplitude at zero momentum transfer. The matrix elements in eqs. (3.4b) and (3.4c) are some auxiliary amplitudes considered only for determination of g and g' and can be computed at arbitrary point since they are momentum independent

to the leading order of the chiral expansion. So, we do not include the contribution of the so-called tadpole term $(\chi^\dagger U + U^\dagger \chi)_{23}$ to the physical amplitude (3.4a). As is well known the appearance of such a term is a consequence of working with a wrong vacuum solution. This term merely renormalizes the strong effective Lagrangian (beyond the mass shell) and can be absorbed into the meson mass matrix by a suitable $SU(3)_L \times SU(3)_R$ rotation. Thus, the tadpoles do not generate any observable effects for the effective chiral Lagrangian of order p^2 to the first order in G_F [3, 27].

The problem is reduced to the determination of the single parameter g. It can be done by studying an appropriate three-point Green's function (GF) via the sum rules technique. We choose the GF in the form

$$G_\mu(p,q) = i^2 \int \langle 0|T j^5_\mu(x) Q_6(0) j^5(y)|0\rangle e^{ip_2x - ip_1y} dxdy =$$

$$= i^2 \int \langle 0|T j^5_\mu(x) Q_6(y) j^5(0)|0\rangle e^{i(p-\frac{q}{2})x + iqy} dxdy \tag{3.5}$$

where $p_1 = p + q/2$, $p_2 = p - q/2$ and we take $j^5_\mu = \bar{d}\gamma_\mu\gamma_5 u$, $j^5 = \bar{u}\gamma_5 s$ as interpolating operators for pion and kaon fields. Because of dispersion relations, GF (3.5) looks like

$$\frac{\langle j^5_\mu|\pi^+(p_2)\rangle\langle\pi^+(p_2)|Q_6|K^+(p_1)\rangle\langle K^+(p_1)|j^5\rangle}{p_2^2(p_1^2 - m_K^2)} + \frac{R_L}{p_2^2} + \frac{R_R}{p_1^2 - m_K^2} + \dots \tag{3.6}$$

where the ellipsis stands for states without any kaon or pion poles, the quark currents projections on the mesonic states are

$$\langle 0|j^5_\mu|\pi^+(p)\rangle = if_\pi p_\mu, \quad \langle K^+(p)|j^5|0\rangle = -i\frac{f_K m_K^2}{m_s} \tag{3.7}$$

and we work with the massless pion (u-, d-quarks). The representation (3.6) needs a comment. The matter is that to extract information about the matrix element in the first addendum in eq. (3.6) one has to distinguish the resonance contribution from miscellaneous ones. It is possible because, contrary to all other states, the resonance leads to the double pole in the dispersion relation. In representation (3.6) it can be achieved within the kinematics where $(pq) = m_K^2/2$. Setting $(pq) = m_K^2/2$ and multiplying eq. (3.6) by the expression $(p^2 + q^2/4 - m_K^2/2)$ one obtains

$$\frac{\langle j^5_\mu|\pi^+(p_2)\rangle\langle\pi^+(p_2)|Q_6|K^+(p_1)\rangle\langle K^+(p_1)|j^5\rangle}{(p^2 + \frac{q^2}{2} - \frac{m_K^2}{2})} + R_L + R_R + \dots \tag{3.8}$$

where the resonance contribution is explicitly distinguished as a pole term. Eq. (3.8) can now be treated by the standard sum rules technique.

Straightforward calculations give the following asymptotic expansion for the GF

$$G_\mu(p,q) = p_{2\mu} G(p,q) + \dots$$

and

$$G(p,q) = -\frac{3}{2\pi^2}\frac{2(pq)}{p^2}\ln\left(\frac{-p^2}{\mu^2}\right)\langle\bar\psi\psi\rangle + \frac{3}{4\pi^2}\gamma\ln\left(\frac{-p^2}{\mu^2}\right)\langle\bar\psi\psi\rangle + \\ +O(p^{-6}) + (\text{bilocal part}) \tag{3.9}$$

where $\langle\bar u u\rangle = \langle\bar d d\rangle \equiv \langle\bar\psi\psi\rangle$. We work in the leading approximation of the q^2 power expansion and simply put $q^2 = 0$. At the same time we have to keep the first order terms in the scalar product $(pq) = m_K^2/2$ and the strange quark mass m_s and we also have to distinguish the strange quark vacuum condensate from the non-strange one $\gamma = \langle\bar s s\rangle/\langle\bar\psi\psi\rangle - 1 \neq 0$ since this difference represents a parameter of $SU(3)_V$ symmetry breaking. As we see all terms of the zeroth order in m_s cancel themselves in eq. (3.9) in agreement with the chiral structure of the operator Q_6. The bilocal part of OPE does not contribute to the parameter g to the $O(p^{-6})$ order due to a specific non-symmetrical choice of the the interpolating currents in eq. (3.5) [12].

To determine the parameter g we apply the finite energy sum rules technique [28] to the function $G(p^2)(p^2 - m_K^2/2)$ which is defined by the equation

$$G(p^2) = G(p,q)\mid_{q^2=0,\ (pq)=\frac{m_K^2}{2}} . \tag{3.10}$$

The result for the matrix element $\langle\pi^+|Q_6|K^+\rangle$ reads

$$\langle\pi^+|Q_6(s_0)|K^+\rangle = \frac{3}{2\pi^2}\frac{m_s\langle\bar\psi\psi\rangle}{f_K f_\pi} s_0\left(1 + \frac{s_0\gamma}{4m_K^2}\right) \tag{3.11}$$

where s_0 is a duality interval. Combining eq. (3.11) with the representation (3.4b) and using the $PCAC$ relation $-2m_s\langle\bar\psi\psi\rangle = f_K^2 m_K^2$ we find

$$g + 2g' = -\frac{3}{8\pi^2}\frac{f_K}{f\pi}\frac{s_0}{f_\pi^2}\left(1 + \frac{s_0\gamma}{4m_K^2}\right). \tag{3.12}$$

It can be easy shown [12] that the first addendum in brackets on the *rhs* of eq. (3.12) represents exactly the contribution of the constant g to the entire sum. Thus we have

$$g = -\frac{3}{8\pi^2}\frac{f_K}{f\pi}\frac{s_0}{f_\pi^2}. \tag{3.13}$$

The question now is which value for the duality interval s_0 has to be used. Throughout the calculation we keep only the first order quantities with respect to $SU(3)_V$ symmetry breaking parameter – the strange quark mass; in eq. (3.11) the chiral suppression is present as a factor m_s. Hence, for consistency, we should use for the duality interval s_0 its value in the chiral limit, *i.e.* the value of the pion duality interval $s_0^\pi = 0.8\ GeV^2$ [29]. There exists a prejudice that the duality interval in the pseudoscalar channel can be abnormally large due to a possible contribution of so-called "direct" instantons [30] but the real estimate of that contribution is practically absent while the use of the above value reproduces the pion decay constant f_π with reasonable accuracy. Actual value of the s_0 parameter can be found only after adding the non-perturbative corrections due to higher dimension operators which are supposed to be small in our case. Finally for the parameter B_p we obtain

$$B_p(s_0^\pi) = \frac{3}{2\pi^2}\frac{s_0^\pi}{f_\pi^2} = 7. \tag{3.14}$$

Eq. (3.14) is written for the normalization point $\mu^2 = s_0^\pi$. For an arbitrary normalization point we have

$$B_p(\mu^2) = B_p(s_0^\pi)\left(\frac{\alpha(s_0^\pi)}{\alpha(\mu^2)}\right)^{-\gamma_6/2\beta_3} \tag{3.15}$$

where $\gamma_6 = -14$ is the anomalous dimension of the operator Q_6.

The strong dependence of the *rhs* of eq. (3.14) on the parameter s_0 is considerably smoothed by the renormalization group factor $(\frac{\alpha(s_0)}{\alpha(\mu^2)})^{7/9}$. Indeed, the result for the parameter $B_p(1\ GeV)$ changes only from 8.19 to 9.38, *i.e.* less than 15%, when one substitutes the kaon duality interval $s_0^K = 1.2\ GeV^2$ [29] into eqs. (3.14, 3.15) (the QCD scale is chosen to be $\Lambda_{QCD} = 0.3\ GeV$).

Let us now estimate the uncertainty of our result. On the physical side the errors originate from the higher order terms in chiral expansion which are omitted in eq. (3.3).

Their relative weight can be represented by the ratio m_K^2/Λ_χ^2, where Λ_χ is a chiral scale parameter. Both on theoretical and empirical grounds one expects [31]

$$\Lambda_\chi^2 = 8\pi^2 f_\pi^2 \sim 1\ GeV^2. \tag{3.16}$$

Thus, a choice of the representation for a quark-gluon operator (for example, Q_6) to the leading order in the chiral effective theory might introduce a 25% error in the estimate of the real physical matrix element.

On the theoretical side of the sum rules the errors come from several sources:

1. Corrections due to operators with higher dimensionality which start with the term $\langle\bar\psi g_s^2 G^2 \psi\rangle(pq)/p^6$ and, probably, some $O(p^{-6})$ bilocal contribution. They are under control and do not lead to any sizable variation of our results though their actual magnitude is questionable because it requires making some estimates of the vacuum expectation values of higher dimension and bilocal operator;
2. Perturbation theory corrections to the coefficient functions of the leading operators. The correct inclusion of these correction requires a complete generalization of the effective Hamiltonian up to the α_s^2 order. Therefore we consequently work only with the first order in α_s terms in the effective Hamiltonian and omit radiative correction to the non-leading operator Q_6 since its coefficient function is already of the α_s order.

Thus, we have calculated the matrix element of the penguin operator via the *QCD* sum rules with the result

$$\frac{1}{i}\langle\pi\pi|Q_6(1\ GeV)|K^0\rangle = -(0.34 \pm 0.09)\ GeV^3 \tag{3.17}$$

or, in terms of the parameter B_p

$$B_p(1\ GeV) = 8.2 \pm 2.1 \tag{3.18a}$$

where the error bars estimate contributions of higher orders of the chiral expansion. The analysis carried out above shows that the uncertainties coming from other sources are relatively small. This result can be important for the precise calculation of the ϵ' parameter.

The value (3.18a) is somewhat larger (with a factor 1.3 when $m_s(1\ GeV) = 175\ MeV$) than the leading order result obtained within the $1/N_c$ expansion framework [5]

$$B_p(1\ GeV) = 6.4 \left(\frac{175\ MeV}{m_s(1\ GeV)}\right)^2 . \tag{3.18b}$$

However a sizable enhancement (factor $1.5 - 2.0$) of the leading order result (eq. (3.18b)) due to the next-to-leading corrections in the $1/N_c$ expansion has been found [14]. This observation is in good agreement with our calculations. Our result gets also into the interval given by lattice models [13]

$$B_p(1\ GeV) = 11 \pm 3 \tag{3.18c}$$

but it is somewhat smaller than optimistic estimate of ref. [7] where the soft pion technique was used

$$B_p(1\ GeV) = 12 \left(\frac{175\ MeV}{m_s(1\ GeV)}\right)^2 . \tag{3.18d}$$

It has been also shown that the Wilson coefficient of the operator Q_6 is essentially increased by the next-to-leading α_s terms [2]. For example, at the point $\Lambda_{QCD} = 0.3\ GeV$, $\mu = 0.8\ GeV$, the numerical value of the leading order coefficient $z_6^{(l.o.)} = -0.028$ while inclusion of the next-to-leading α_s corrections leads to the result $z_6^{(n.l.)} = -0.098$. However the consistent generalization of the effective Hamiltonian beyond the leading logarithmic approximation requires to derive the corresponding mesonic matrix elements up to the same order in α_s (see Sect. 2). Thus this enhancement in general can be canceled by radiative corrections to the matrix element of the operator Q_6. If it does not occur the penguin operator becomes quite important in the analysis of $\Delta I = 1/2$ rule since it provides approximately 20% of the physical value of the decay amplitude with the isotopic spin transfer $\Delta I = 1/2$.

4 The scalar gluonium contribution to the $K \to \pi\pi$ decay amplitude.

As it has been already pointed out the possible source of the enhancement of the $\Delta I = 1/2$ amplitude is the unfactorizable contributions to the matrix elements of the local four-

quark operators produced by the low-energy strong interaction of the light quarks. In the chiral Lagrangians approach these non-factorizable contributions reveal themselves in two different ways: first, they appear as corrections to the couplings characterizing the "factorizable" weak chiral Lagrangian in $O(p^2)$ and higher orders, second, some new non-factorizable terms emerge. The latter is the case for $K \to \pi\pi$ decay mode with gluons forming an intermediate state.

The unfactorizable corrections of the first and of the second type have already been discussed in the framework of $1/N_c$ expansion [4, 5]. In ref. [4] the non-perturbative corrections to the weak chiral Lagrangian of the leading $O(p^2)$ order caused by the presence of the non-perturbative gluon condensate was derived while in ref. [5] the $O(p^4)$ order contribution of the unfactorizable chiral loops was computed and the sizable violation of the factorization has been found. However in both pointed approaches the above mode was omitted. Meanwhile it is quite reasonable to suppose that the gluons being considered as exchange particles can be important in explanation of the $\Delta I = 1/2$ rule because they carry a zero isospin and contribute only to the $\Delta I = 1/2$ amplitude. Moreover that kind of contribution being non-leading in chiral expansion nevertheless can be sizable due to the strong effects of the non-perturbative gluon vacuum.

Before treating the long-distance effects of the meson-gluon transitions it is very useful to consider the similar phenomenon arising already in perturbative QCD as a leading correction in the inverse mass of the charmed quark given by eq. (2.3). If we restrict the analysis to the scalar colorless gluon configuration $G^a_{\mu\nu}G^a_{\mu\nu}$ only the operators $Q_3^{(8)}$, $m_s Q_1^{(7)}$, and $m_s Q_2^{(7)}$ give a contribution and eq. (2.3) gets the form

$$H_G^{(8)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (1-\tau) \left(-\frac{1}{120} \frac{1}{m_c^2} m_s \bar{s}_R d_L \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \right) + h.c. \tag{4.1}$$

To derive the effective chiral Lagrangian involving Goldstone degrees of freedom only which then can be used for the calculation of the decay amplitude one has to replace the QCD operator

$$m_s \bar{s}_R d_L \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \tag{4.2}$$

in eq. (4.1) by its mesonic realization. Using its chiral transformation properties one can

write down the representation

$$m_s \bar{s}_R d_L \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} = A f_\pi^6 (U^\dagger \chi)_{23} + B f_\pi^4 (U^\dagger \chi)_{23} tr_{fl}(\partial_\mu U^\dagger \partial^\mu U) +$$
$$+ (\text{other } O(p^4) \text{ terms}) + O(p^6) \tag{4.3}$$

where A and B are the dimensionless parameters. The second term on the *rhs* of eq. (4.3) is separated from the other $O(p^4)$ structures for the reason will be clarified below. The $O(p^2)$ term in eq. (4.3) is exactly a tadpole term which should be omitted. So the contribution to the physical amplitude is determined by the $O(p^4)$ part of the chiral representation (4.3). The most transparent way to obtain this part is to consider the quark-gluon operator (4.2) as a product of the (pseudo)scalar quark current and scalar colorless gluon operator. Then one can replace the quark current by its mesonic realization according to the $PCAC$ hypothesis and current algebra [26]

$$m_s \bar{s}_R d_L \to -\frac{f_\pi^2}{8} (U^\dagger \chi)_{23}. \tag{4.4}$$

On the other hand there is a low-energy theorem based on the fundamental properties of the energy-momentum tensor which gives the chiral representation of the gluon operator [32, 33]

$$\frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} = -\frac{2}{\beta_3} f_\pi^2 tr_{fl}(\partial_\mu U^\dagger \partial^\mu U) + O(p^4). \tag{4.5}$$

Eqs. (4.4, 4.5) allow us to determine the parameter B

$$B = \frac{1}{4\beta_3} \tag{4.6}$$

which now modulates the unique term of $O(p^4)$ order in eq. (4.3). This approximation corresponds to the simplest physical picture where the kaon is annihilated by the pseudoscalar quark current while the pion pair is born by the gluon operator. We should not that the above separation of the operator (4.2) into the quark and gluon parts leads to some uncertainties in the final result since the effects of the interaction between them are lost. However the corresponding corrections seem to be suppressed at least at the perturbative level. Eqs. (4.1, 4.3-4.6) result in the effective chiral Lagrangian of the form

$$L_G^{ch} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* (1-\tau) \left(-\frac{1}{480\beta_3} \frac{f_\pi^4}{m_c^2} (U^\dagger \chi)_{23} tr_{fl}(\partial_\mu U^\dagger \partial^\mu U) \right) + h.c. \tag{4.7}$$

Since the pion pair is born by the gluon operator this Lagrangian describes the investigated decay channel with gluons forming an intermediate state.

Now the corresponding $K \to \pi\pi$ decay amplitude becomes explicitly calculable. We will use the standard parametrization of the amplitude A_0 with the isospin transfer $\Delta I = 1/2$

$$ReA_0 = \frac{G_F}{\sqrt{2}} \sin\theta_c \cos\theta_c g_{1/2} f_K m_K^2 \tag{4.8}$$

where θ_c stands for Cabibbo angle, $g_{1/2}$ is a dimensionless parameter. The new contribution reads

$$\Delta g_{1/2} = \frac{1}{30\beta_3} \frac{m_K^2}{m_c^2} \sim 10^{-3}. \tag{4.9}$$

While the experiment gives [19]

$$g_{1/2}^{exp} = 3.9 \tag{4.10}$$

and the most recent theoretical estimation is $g_{1/2} \sim 2.4$ [2]. As we see the local (perturbative) part of the new decay mode is negligible according to the general estimate of the scale of the leading order charmed quark mass corrections done in Sect. 2.

However the local effective Hamiltonian (4.1) does not exhaust the whole physics of the meson-gluon transitions. It cannot account for the long-distance contribution connected with the propagation of the soft u-quark round the loop of the annihilation diagram. Because of the lightness of the u-quark this contribution can not be represented as a local vertex. It ultimately depends on the infrared properties of QCD and requires non-perturbative approach. We will follow the general line of the approach applied in Sect. 3 to the calculation of the penguin operator matrix element.

In so doing we start with a tree level Hamiltonian which after decoupling of the c-quark has the form

$$H^{tr}_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} Q_2 + h.c. \tag{4.11}$$

The quantity of interest is an effective theory realization in terms of mesonic variables of the part of the operator $Q_2 \equiv Q_2^u$ which is responsible for the kaon transfer into gluons. Invoking the results of our previous consideration we can write down this part in the form

$$Q_2^G = g^G f_\pi^2 (U^\dagger \chi)_{23} tr_{fl}(\partial_\mu U^\dagger \partial^\mu U) \tag{4.12}$$

where g^G is a dimensionless parameter to be computed. We should note that the chiral representation of the whole operator Q_2 contains a large number of structures but we are interested only in the part corresponding to the transition with the gluons forming an intermediate state which has the unique representation (4.12). Indeed, there is the single $SU_V(3)$ octet terms (4.12) in the chiral weak Lagrangian describing the $K \to \pi\pi$ decays which is proportional to the $tr_{fl}(\partial_\mu U^\dagger \partial^\mu U)$ as it required by eq. (4.5) [3]. Thus the problem is reduced to the computation of the chiral coupling constant g^G. As for the parameters g and g' of the chiral representation of the penguin operator it can be done by studying the appropriate GF via QCD sum rules technique. For the technical reason working with a two point GF is preferable. In the given decay channel the pions are born by a gluon cloud therefore the gluon operator $G^a_{\mu\nu}G^a_{\mu\nu}$ can play the role of an interpolating operator of the pion pair. Thus, it is natural to choose GF in the form

$$G(p) = \int \langle 0|T\frac{\alpha_s}{\pi}G^a_{\mu\nu}G^a_{\mu\nu}(x)Q_2(0)|K^0(q)\rangle e^{ipx}dx|_{q=0}. \qquad (4.13)$$

A remark about the chiral limit for the kaon in eq. (4.13) is necessary. The representation (4.12) fixes the correct $O(p^4)$ chiral behavior of the considered decay amplitude and does not depend explicitly on the kaon momentum. Keeping a non-vanishing kaon momentum leads to a shift of the decay amplitude that lies beyond the accuracy of the present approach. Thus we can put $q = 0$ in eq. (4.13) and work with GF depending on one argument only.

Saturating GF (4.13) by the $\pi^+\pi^-$ and $\pi^0\pi^0$ states (the lowest states with the proper quantum numbers), substituting the operator Q_2 for its mesonic realization (4.12) and using the low-energy theorem (4.5) one obtains at small momentum p the following physical representation

$$G(p) = g^G\frac{32}{\pi^2\beta_3}\frac{m_K^2}{f\pi}p^4\ln\left(\frac{-p^2}{\mu^2}\right) + O(p^6). \qquad (4.14)$$

The theoretical side reads after making use of OPE

$$G(p) = i\frac{1}{2\pi^2}\ln\left(\frac{-p^2}{\mu^2}\right)\frac{\alpha_s}{\pi}\langle 0|m_s\bar{s}_R g_s G^a_{\mu\nu}t^a\sigma_{\mu\nu}d_L|K^0(q)\rangle|_{q=0} + O(\alpha^2p^2) + O(p^{-2}). \quad (4.15)$$

Factor m_s in eq. (4.15) provides the correct chiral property of GF and justified the representation (4.12) for the operator Q_2. By contraction of the kaon state one can transform eq. (4.15) into the expression

$$G(p) = \frac{1}{4\pi^2} \ln\left(\frac{-p^2}{\mu^2}\right) \frac{\alpha_s}{\pi} f_K m_K^2 m_0^2. \tag{4.16}$$

For extracting information about the chiral coupling constant g^G we use finite energy sum rules with the result

$$g^G = \frac{3\beta_3}{128} \frac{f_K}{f_\pi} \frac{f_\pi^2 m_0^2}{s_0^2} \frac{\alpha_s(s_0)}{\pi}. \tag{4.17}$$

To take into account the strong interaction at short distances the operator Q_2^G in the effective Hamiltonian has to be multiplied by the corresponding Wilson coefficient $z_2(s_0)$. Finally the new contribution to the theoretical estimate of the $\Delta I = 1/2$ amplitude in terms of parameter $g_{1/2}$ takes the form

$$\Delta g_{1/2} = z_2(s_0) \frac{3\beta_3}{8} \frac{m_0^2 m_K^2}{s_0^2} \frac{\alpha_s(s_0)}{\pi}. \tag{4.18}$$

This result needs some comments.

1. This next-to-leading in $1/N_c$ expansion contribution is missed within the factorization framework and also within any approach where quark currents in four-quark operators are replaced by their mesonic counterparts separately.

2. The gluon cloud in the intermediate state does not form a resonance state and, therefore, the contribution (4.18) is not suppressed by a large scalar meson mass.

3. In general the more complicated scalar colorless gluon configuration, for example $f^{abc} G^a_{\mu\nu} G^b_{\nu\lambda} G^c_{\lambda\mu}$, could be an intermediate state in this channel as well. However the theorem (4.5) shows that the two pion form factor of these configurations can be of $O(p^4)$ or higher order in chiral expansion and that lead only to the negligible $O(p^6)$ shift of the decay amplitude. Indeed, the theorem (4.5) sets the equivalence between the trace of the energy momentum tensor expressed in terms of QCD and mesonic degrees of freedom. In chiral limit in the leading order in momentum expansion there is a single Lorentz invariant flavor singlet mesonic configuration which is proportional to the trace of the

energy momentum tensor. On the other hand there are no extra gluon contributions to the conform anomaly.

The question now is what numerical value for the duality interval s_0 has to be used. Actually, the allowed value of the duality threshold is quite restricted by the form of the physical spectrum and by the requirement of absence of uncontrollable α_s corrections. To suppress contributions of higher mass states, for example, a scalar meson $\sigma(0.9\ GeV)$, to the considered channel one has to take $s_0 < (0.9\ GeV)^2$. At the same time the physical representation (4.14) is obtained in the leading order in chiral perturbation theory and the whole procedure is justified until the ratio s_0/Λ_χ^2 remains small. On the other hand at the scale μ less than $0.8\ GeV$ the perturbative α_s corrections to Wilson coefficients become uncontrollable [2] and for consistency of the approach one has to set the low limit of the duality interval to be $s_0 > (0.8\ GeV)^2$. The reasonable choice for the duality interval now reads $s_0 = (0.8\ GeV)^2$.

Following the approach of the Sect. 3 let us estimate the uncertainty of our result. On the physical side of sum rules the errors related to higher order terms in chiral expansion, which have been omitted in eq. (4.12), are, in general, unknown. But one can hope that in the spirit of chiral perturbation theory they are about 25% [26, 31]. On the theoretical side of the sum rules the errors come from two sources. The first one is the perturbative part of *OPE* (the unit operator) that is suppressed by a loop factor $\alpha_s/4\pi \sim 10^{-3}$ and cannot lead to a sizable change of our result. Next non-perturbative corrections due to operators with higher dimensionality seem to be more important. They start with the dimension eight operators which have already been discussed. Numerical estimates would require knowing the matrix elements of those operators between the kaon and the vacuum state which are not available now. But as a first approximation the relative weight of these corrections can be represented by the ratio

$$\frac{\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle}{\Gamma(5)} : \frac{s_0 m_0^2}{4\pi^2 \Gamma(3)} \sim 0.1 \tag{4.19}$$

where $\langle \frac{\alpha_s}{\pi} G^a_{\mu\nu} G^a_{\mu\nu} \rangle \sim (330\ MeV)^4$ [34], the Gamma function factor $1/\Gamma(n)$ comes from the quark loop with the $n-1$ gluon field or mass insertion. Here we find the same

situation as in the analysis of $1/m_c$ corrections where the contribution of the dimension eight operators is suppressed rather numerically than parametrically.

Taking into account the uncertainty of determination of the parameter m_0^2 we estimate the error bound to be about 40%. Numerically one obtains

$$\Delta g_{1/2} = 0.56 \pm 0.22 \tag{4.20}$$

at the point $s_0 = \mu^2 = (0.8\ GeV)^2,\ \Lambda_{QCD} = 300\ MeV,\ z_2(s_0) = 1.49$ [9].

Thus, the new contribution provides about 15% of the experimentally observable amplitude (4.10). At the same time it is comparable with the leading order result for the decay amplitude obtained by the naive factorization of the four-quark operator Q_2 when all strong interaction corrections are neglected

$$g_{1/2}^{fac} = 5/9. \tag{4.21}$$

This implies the strong violation of factorization in the $O(p^4)$ order in chiral expansion that leads to additional enhancement of the theoretical estimate of the $K \to \pi\pi$ decay amplitude with the isospin transfer $\Delta I = 1/2$.

5 Conclusion.

In the present paper we make an attempt to clarify the role of annihilation or "penguin" mode in the descriptions of the $K \to \pi\pi$ decay in the Standard Model. We concentrate our attention on some new aspects which have not been considered yet.

The complete form of the effective low-energy $\Delta S = 1$ Hamiltonian up to the first order in m_s, $1/m_c$ and α_s with $m_t \sim M_W$ is obtained. The violation of factorization for the mesonic matrix element of the operator Q_2 caused by $K \to \pi\pi$ transition with the gluons in the intermediate state is also analyzed.

We propose the regular method to evaluate the mesonic matrix elements of the effective Hamiltonian for $K \to \pi\pi$ transitions. The weak Hamiltonian is represented through the variables of chiral effective theory and the coefficients of proportionality between quark-gluon operators and mesonic operators are determined via QCD sum rules for an

appropriate three-point GF. We apply this technique for computing the QCD penguin operator matrix element.

The main results are

1. The contribution of the dimension eight operators (non-leading $1/m_c$ corrections) proves to be small numerically (about 10 % of the leading term) when one estimates the corresponding mesonic matrix element within the simplest factorization framework. However its real magnitude can be estimated only when more detailed information on the matrix elements of the local operators with dimension eight becomes available, for example, from lattice calculations.

2. The quark-gluon operator $m_s \bar{s}_R g_s G_{\mu\nu} \sigma^{\mu\nu} d_L$ contributes to the imaginary part of the effective Hamiltonian in the case of a heavy top quark already in the first order in α_s due to incomplete GIM cancelation. However its contribution to the parameter ϵ' is suppressed because of the smallness of the corresponding Wilson coefficient and the specific (tadpole) chiral structure of this operator.

3. Sum rules predict for the matrix element of the QCD penguin operator

$$\frac{1}{i}\langle \pi\pi | Q_6(1\ GeV) | K^0 \rangle = -(0.34 \pm 0.09)\ GeV^3$$

where the error bars estimate contributions of higher orders of the chiral expansion. The analysis carried out above shows that the uncertainties coming from other sources are relatively small. This value is somewhat larger than the results obtained within the $1/N_c$ approximation framework [4, 5] but it is somewhat smaller than optimistic estimate of ref. [7]. Our result gets also into the interval given by lattice models [13]. The relatively large value of the matrix element along with the strong enhancement of the corresponding Wilson coefficient [2] make the penguin operator quite important in the analysis of the $\Delta I = 1/2$ rule since it provides about 20% of the decay amplitude with $\Delta I = 1/2$.

4. The non-perturbative contribution to the mesonic matrix element of the leading operator Q_2 produced by the transitions with gluons in the intermediate state gives an additional enhancement of the theoretically predicted $K \to \pi\pi$ decay amplitude with $\Delta I = 1/2$ and provides about 15% of the experimentally observable amplitude value. New

contribution is of $O(p^4)$ and is lost within the factorization framework but numerically is comparable with the result of naive factorization for the $\Delta I = 1/2$ decay amplitude. It allows us to conclude that there is a sizable violation of the factorization in the $O(p^4)$ order in chiral expansion.

To conclude, the comparison between the "penguin-like" and "non-penguin" parts of the $\Delta I = 1/2$ amplitude is quite instructive. According to the recent work [2] the current-current operators Q_1 and Q_2 (without Q_2^G part) give about 45% of the experimentally observable amplitude while the total effect of the "penguin-like" contributions of the operators Q_6 and Q_2^G turns out to be about $30 - 40\%$. So large "penguin-like" contribution improves considerably the theoretical description of the $\Delta I = 1/2$ rule in non-leptonic kaon decays. Indeed, our results imply $ReA_0^{th} \sim 0.8 ReA_0^{exp}$ while ref. [2] gives $ReA_0^{th} \sim 0.6 ReA_0^{exp}$ with penguin contribution being only about 15%. As we see, the discrepancy between theory and experiment still exists but it is getting smaller. This result can be presented in more impressive form if one considers the ratio ReA_0/ReA_2 where A_2 stands for the amplitude with the isospin transfer $\Delta I = 3/2$ [2]. Then we obtain $(ReA_0/ReA_2)^{th} \sim 21.$ that is very close to the experimental data $(ReA_0/ReA_2)^{exp} = 22.3$. We should emphasize that this result is obtained without the standard practice of using the extremely low normalization point or extremely light strange quark. Since the resources of the perturbation theory seem to be exhausted the further progress in the theoretical explanation of the $\Delta I = 1/2$ problem will probably be connected with studying the unfactorizable contributions to the mesonic matrix elements caused by strong interactions at low energy.

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